Assignment 5

due Wednesday, November 15, 2006

1. Exercise 13.1-6, p 277. [6 points]

2. Give a binary tree which can be colored as a red-black tree but which cannot be an AVL tree. [4 points]

3. Do (i) exercise 13.3-2, p 287, (ii) then insert 21, 28, and 30, (iii) and finally delete 41. Show the tree after each phase (and more if you wish). [8 points]

4. Let T be a tree storing 200,000 items. What is the worst case height of T in the following cases?
   (a) T is an AVL tree
   (b) T is a (2,4) tree
   (c) T is a red-black tree
   (d) T is a binary search tree [8 points]

5. Exercise 18.2-1, p 447, but change the minimum degree from 2 (which would be a (2,4)-tree) to 3. [6 points]

6. Let $T$ and $U$ be two red-black trees storing $n$ and $m$ items, respectively, such that any item in $T$ has a key less than the keys of all items in $U$. Describe an $O(\lg n + \lg m)$ method for joining the trees into a single tree that stores all the items in $T$ and $U$. The original $T$ and $U$ may be be destroyed in the process. [8 points]

7. Show how to use a heap to find the $k^{th}$ smallest of a set of $n$ elements in $O(n + k \log n)$ time [6 points]

Total: 46 points

Notes:

- Q1: As usual, internal node means actual node and external node means a null.
- Q2: Of course, here we are just considering the shape of the tree. You are to show a tree which could be colored as a RB tree but is too out of balance to be an AVL tree.
- Q6: Try to adapt the solution of the same (2,4)-tree problem.
- Q7: Use the fact that heap-build is $O(n)$ and heap-delete is $O(\log n)$. And of course you will want to use a min-heap instead of a max-heap.