Assignment 3

due Wednesday, February 2, 2005

1. Draw the binary tree whose inorder traversal is \textit{abcdefghijkm} and whose postorder traversal is \textit{acedbfihkmljg}. [5 points]

2. The \textbf{balance factor} of an internal node $v$ of a binary tree is the difference between the heights of the left and right subtrees of $v$. Write a recursive routine which will print the balance factors of all nodes in a binary tree. [6 points]

3. Consider an ordered tree $T$ and a binary tree $T'$ representing it, using the first-child next-sibling representation (section 10.4). An inorder traversal of $T'$ is equivalent to what kind of traversal of $T$? [4 points]

4. In class we defined the internal path length $I$ and the external path length $E$, both measures of a binary tree. If that tree has $n$ nodes, show that $E = I + 2n$. [8 points]

5. Consider the tree of Figure 12.2 on p 257. How many different permutations of the values it contains, when inserted in that order, will yield this particular tree? [8 points]

6. How many permutations of \textit{1, 2, \ldots, n} yield a skew tree? (Since any one skew tree is generated by just one permutation, this question is asking for the number of skew trees of $n$ nodes.) [5 points]

7. \textit{(Search path splitting a BST)} Exercise 12.2-4, p 260. [4 points]

Total: 40 points

Notes:

\begin{itemize}
\item \textit{(Q3)} To get $T'$, imagine the first-child as a left pointer and the next-sibling as a right pointer.
\item \textit{(Q4)} We had $I = \sum_{v \in V} d(v)$, where $V$ is the set of nodes and $d(v)$ is the depth of a node. $E$ is defined similarly; over all external nodes. You will want to use induction.
\item \textit{(Q5)} Consider a tree where
  \begin{itemize}
  \item the left subtree contains $n$ nodes and is generated by $r$ permutations
  \item the right subtree contains $m$ nodes and is generated by $s$ permutations
  \end{itemize}
  Then the whole tree contains $n + m + 1$ nodes and is generated by $r \cdot s \cdot \binom{n+m}{n}$ permutations.
\end{itemize}