Assignment 4

due Wednesday, June 1, 2005

1. Give a resolution refutation which shows the correctness of the following inference:

If John went swimming, then he lost his glasses and did not go to the movies. If John ate too much meat and did not go to the movies, then he will suffer indigestion. Therefore, if John ate too much meat and went swimming, then he will suffer indigestion.

2. Let us be given a plane map containing infinitely many countries. Suppose there is no way to color this map with $k$ colors so that adjacent countries are colored with different colors. Prove there is a finite submap for which the same is true.

3. For each of the following sentences, give an interpretation which is a model as well as one which is not.

\( (\forall x)(\exists y)(\forall z)(s(x, c) \Rightarrow r(x, y, z)) \)
\( (\exists y)(\forall x)(\forall z)(s(x, c) \Rightarrow r(x, y, z)) \)
\( (\forall x)(\exists y)(s(x, y) \Rightarrow s(y, x)) \)

4. Give an interpretation which is a model of 3(a) but not 3(b),

5. Consider the inference

\( (\forall x)(p(x) \Rightarrow (\forall y)(s(y, x) \Rightarrow u(x))), \quad (\exists x)(p(x) \land (\exists y)(s(y, x) \land h(y, x))) \)
\n\( \models (\exists x)(\exists y)(u(x) \land h(y, x) \land s(y, x)). \)

(a) Find a universal sentence whose unsatisfiability is equivalent to the correctness of this inference. The remainder of this problem will be easier if you use generalized Skolem-ization to introduce only constant symbols.

(b) Describe the Herbrand universe and base.

(c) Show that the inference is correct.

6. Let $\Omega_1, \Omega_2$ be sets of sentences such that $\Omega_1 \cup \Omega_2$ is unsatisfiable. Prove that there is a sentence $\alpha$ such that $\Omega_1 \models \alpha$ and $\Omega_2 \models \neg \alpha$. [Hint: Use the compactness theorem for the predicate calculus.]

7. exercise 1, p 403

8. Prove the correctness of the formula of exercise 3, p 387

9. exercise 4, p 403

10. exercise 3, p 410