Assignment 2

due Wednesday, April 27, 2005

1. Show that there is no computable function $f$ such that $f(x) = \Phi(x, x) + 1$ whenever $\Phi(x, x) \downarrow$.

2. Let $g(x)$ and $h(x)$ be partially computable functions. Show that there is a partially computable $f(x)$ such that $f(x) \downarrow$ for exactly those values $x$ for which either $g(x) \downarrow$ or $h(x) \downarrow$ (or both) and such that when $f(x) \downarrow$ either $f(x) = g(x)$ or $f(x) = h(x)$.

3. Can $f$ be found satisfying all the requirements of the previous problem but such that in addition $f(x) = g(x)$ whenever $g(x) \downarrow$? \textit{Hint:} use the previously previous problem.

4. Prove that there is a primitive recursive function $h(u)$ such that for all $x, u$,

$$\Phi(x, h(u)) = \Phi(x, \Phi(h(u), u)).$$

\textit{Hint:} apply the recursion theorem to $g(z, x, u) = \Phi(x, \Phi(S^1_1(u, z), u))$ and then use the parameter (aka s-m-n) theorem.

5. Write LOOP programs for the following functions. Actually write them out, rather than just showing that they exist.

   (a) Show $f_2(x) = 2^x$ is in $L_2$.

   (b) Show $f_3(x) = 2^x^2$ is in $L_3$.

6. Show that $x \cdot y$ is in $L_2 - L_1$