Main topics of the week:
- Formal Definition of Grammar
- BNF Notation
- Languages and Grammars
- Parse Trees
- Ambiguity in Grammars
- Attribute Grammars
- Dynamic Semantics
- Parsing

Before we look at specific examples of languages, we will first spend some time considering the general structure of languages – that is, how the syntax of a language is formally specified by a grammar.

Syntax and Parsing

A program consists of a string of characters. The compiler and interpreter’s jobs are to determine which of these strings constitute a legal program and arrange for the execution. The process will begin with a lexical analysis. Basically, the lexical analysis will break the string of characters into meaningful substrings. These substrings are technically called the **lexemes** of the language and categories of them are called the **tokens** of the language. They can be things like variable names, keywords, operator symbols, etc. Lexemes cannot be broken into smaller pieces, so they are the atomic units of the language. The job of the lexical analyzer is to convert the program into a sequence of lexemes. This is generally done with pattern matching, and in fact the lexical syntax of the language is a regular grammar. The program that does the lexical analysis is usually called a scanner, and is actually an instance of a deterministic finite automaton. The scanner typically ignores and discards white space and comments in the program source, using white space only to the extent it is necessary to distinguish lexemes.

The next phase in the process is syntactical analysis of the stream of tokens. The job here is to determine if the program is syntactically correct, that is, is it a legal program. This stage is not concerned with any meaning of the program, rather just the form of the program – is it using the legal words of the language, and are they arranged correctly (i.e., is the grammar correct). The result of doing a syntactical analysis is to produce an abstract syntax tree (or parse tree).

Once the **abstract syntax tree** is built, it can be **semantically** analyzed and annotated with the semantic actions associated with the language constructs. This gives meaning to the program and provides the information needed to generate code (for immediate interpretive execution or later static execution).

We won’t focus on lexical analysis – that is a pretty straightforward task done by tools like Unix **lex**. It typically involves regular expressions to specify what character sequences constitute tokens in the language. We are more concerned with how to specify the syntax of a language. Tools are also available for this parsing, notably Unix **yacc**. This tool gets tokens from lex and according to a grammar we specify and code we provide, builds an abstract syntax tree (or whatever else we want to do). We won’t go over the code provided (that would be part of a compiler course), but we do want to
understand how to specify a grammar. A parser like yacc uses recursion and is basically a push down automaton.

**Grammars**

We need to have some way of precisely describing a language so that we can tell if a given program is a legal program in the language. English descriptions could be used to define the language, but are easily subject to misinterpretation. A **context free grammar** (from now on, just grammar) is a formalism for specifying this structure of a language precisely. It specifies how the tokens can be combined to produce legal programs and generally reveals something about the structure of programs in the language. A grammar consists of

1) A set T of terminal symbols
2) A set N of non-terminal symbols (sometimes called variables)
3) A set P of production rules
4) A special start symbol S

Although this is a precise definition of a grammar, it is not to be construed as a programmer’s guide to the language. The grammar is the “legal” definition of the syntax of the language but is not likely to be the place you would look if you were trying to learn the language. However, if you were writing a compiler for the language, you would certainly be interested in seeing this specification of the grammar.

**Backus-Naur Form** (BNF) is a notation used to write down a grammar. The terminals are usually just written as themselves (or sometimes in quotes to emphasize they are literal). Non-terminals are often written between angle brackets and may have suggestive names like `<expr>` or `<statement>`. The production rules consist of a non-terminal on the left, the symbol `::=` (or an arrow: `→`) and the right hand side as a sequence of terminals and non-terminals. Single production rules for the same non-terminal may be combined into a single rule by joining the right hand sides of the rules into a single right hand side with the “or” operation denoted by `|`. Unless otherwise specified, the non-terminal on the left of the first rule is the start symbol. Another style is to use upper case letters for the non-terminals and lower case letters for the terminals.

A grammar gives the rules for producing all legal programs. The way the rules work is this: beginning with the start symbol, we replace it using any of the rules for the start symbol. In the resulting string, any non-terminals are replaced according to a rule for them. In this way, we keep eliminating non-terminals until we have a string consisting of just terminals. This result is called a **production** of the grammar, and is a legal program according to the grammar. The sequence of substitutions using the rules of the grammar is called a **derivation**.

Here are some examples of production rules that you might expect to find in a language parser:

```
<stmt> ::= <var> = <expr> ;
<var> ::= A | B | C
<expr> ::= <var> + <var> | <var> - <var> | <var>
```

This is just a simple production with start symbol `<stmt>` that would generate assignment statements consisting of an identifier name (A, B, or C), the assignment
operator ‘=’, the variable name B, a plus sign, the variable C, and a terminating semicolon. From this little grammar, we could generate ‘A = B + C;’, that is, this statement would be in the language of the grammar. We can see that this statement is in the grammar by the derivation:

\[<\text{stmt}> \rightarrow <\text{var}>=<\text{expr}>; \rightarrow A=<\text{expr}>; \rightarrow A=<\text{var}>+<\text{var}>; \rightarrow A=B+<\text{var}>; \rightarrow A=B+C;\]

The derivation can also be represented by a **parse tree**:

![Parse Tree](image)

where we read the resulting generated string in the language along the bottom. Notice that in a parse tree, the start variable is at the top, and its children are a production rule. All leaves in the parse tree must be terminals and internal nodes are the non-terminals.

Grammars do not have to be unique. The above grammar could add the rules:

\[
<\text{sum}> ::= <\text{var}> + <\text{var}> \\
<\text{diff}> ::= <\text{var}> - <\text{var}>
\]

and change the rule:

\[
<\text{expr}> ::= <\text{sum}> | <\text{diff}> | <\text{var}>
\]

This clearly generates the same language, but the parse trees would be different since there would be more internal nodes in the example given.

A good grammar captures the logical structure of the language, and like “good” programs, uses meaningful names, and is easy to read and as unambiguous as possible.
Here’s a fragment of the top level of a programming language (terminals are indicated by all caps):

\[
\begin{align*}
\text{program} &::= \text{declarations} \quad & \text{declarations} &::= \text{process} \\
& | \text{declarations} \quad & \text{declarations} &::= \text{declarations} \quad & \text{declarations} &::= \text{process} \\
& | \text{declarations} \quad & \text{declarations} &::= \text{process} \\
& | \text{declarations} \quad & \text{process} &::= \text{SESSION ID body} \\
& | \text{declarations} \quad & \text{process} &::= \text{SESSION ID LPAREN arg_list RPAREN body} \\
& | \text{declarations} \quad & \text{process} &::= \text{SUBSESSION dtype ID LPAREN param_list RPAREN body} \\
\text{declarations} &::= \text{declarations} \quad & \text{declarations} &::= \text{process} \\
& | \text{declarations} \quad & \text{declarations} &::= \text{declarations} \\
\text{process} &::= \text{SESSION ID body} \\
& | \text{SESSION ID LPAREN arg_list RPAREN body} \\
& | \text{SESSION ID LPAREN arg_list RPAREN body} \\
\text{body} &::= \text{LBRACE statement_list RBRACE} \\
\text{statement} &::= \text{expr SEMI} \\
& | \text{SEMI} \\
& | \text{compound statement} \\
& | \text{PRINT expr SEMI} \\
& | \text{IF LPAREN expr RPAREN statement} \\
& | \text{IF LPAREN expr RPAREN statement ELSE statement} \\
& | \text{SWITCH LPAREN expr RPAREN LBRACE case_list RBRACE} \\
& | \text{FOR LPAREN expr SEMI expr SEMI expr RPAREN statement} \\
& | \text{WHILE LPAREN expr SEMI expr RPAREN statement} \\
& | \text{CONTINUE SEMI} \\
& | \text{BREAK SEMI} \\
& | \text{EXIT SEMI} \\
& | \text{EXIT LPAREN RPAREN SEMI} \\
& | \text{EXIT expr SEMI} \\
& | \text{REGION ID cstatement} \\
& | \text{SESSION_RETURN expr SEMI} \\
& | \text{SESSION_RETURN SEMI} \\
& | \text{TRY cstatement catch_list FINALLY cstatement} \\
& | \text{TRY cstatement catch_list} \\
& | \text{TRY cstatement catch_list} \\
& | \text{THROW ID SEMI} \\
& | \text{THROW ID LPAREN expr RPAREN SEMI} \\
& | \text{THROW ID LPAREN expr RPAREN SEMI} \\
\end{align*}
\]
Grammar Ambiguity

Consider the following small grammar for assignment statements:

\[
\text{<assign>} ::= \text{id} = \text{<expr>}
\]
\[
\text{id} ::= A | B | C
\]
\[
\text{<expr>} ::= \text{<expr>} + \text{<expr>} | \text{<expr>} * \text{<expr>} | ( \text{<expr>} ) | \text{id}
\]

The sentence \( A = B + C * A \) has two different parse trees:

This happens since the grammar allows \text{<expr>} to grow on the left or the right. This ambiguity can be a problem for semantic analysis if the semantic analysis is based on the parse tree, which it often is. This particular example is a case where we want the grammar to reflect operator precedence. That is, multiplication should bind more tightly than addition to follow the usual rules in algebra. We can cause this to happen in the grammar by introducing more symbols:

\[
\text{<assign>} ::= \text{id} = \text{<expr>}
\]
\[
\text{id} ::= A | B | C
\]
\[
\text{<expr>} ::= \text{<expr>} + \text{<term>} | \text{<term>}
\]
\[
\text{<term>} ::= \text{<term>} * \text{<factor>} | \text{<factor>}
\]
\[
\text{<factor>} ::= ( \text{<expr>} ) | \text{id}
\]

Then we will have a unique parse tree for \( A = B + C * A \), and the parse tree reflects the higher precedence of multiplication. Inserting parentheses alters this precedence as expected, because our grammar has parentheses enclosing just an \text{<expr>}, bringing us back to an expression subtree. This grammar also gives us left associativity of the operators since the parse trees expand to the left for + or *. Generally, when a production rule has its LHS also appearing at the beginning of the RHS, the rule is called **left recursive**, and this captures the idea of left associativity. If the LHS appears at the end of the RHS, then we have a right recursive rule, which implements right associativity.

Ambiguity can cause problems for statements as well, and the classic example is the dangling else. Suppose we have the grammar:

\[
\text{<if-stmt>} ::= \text{if <expr> <stmt>} \mid \text{if <expr> <stmt> else <stmt>}
\]
\[
\text{<stmt>} ::= \text{<if-stmt>} \mid \text{S1} \mid \text{S2}
\]
For the sentence “if <expr> if <expr> S1 else S2” we would get two distinct trees:

```
<if-stmt>  
  <stmt>    
  <if-stmt>  
  if <expr> if <expr> S1 else S2
```

```
<if-stmt>  
  <stmt>    
  <stmt>    
  if <expr> if <expr> S1 else S2
```

and this clearly poses a problem for semantic analysis: who does the else belong to? It can clearly make a difference in the operation of the program since a false first expression and true second expression results in the execution of S1 with the first tree, but in the execution of no statement with the second tree. To fix this, we would need additional non-terminals to distinguish between else-less if’s and if-else’s or change the language to include delimiters. For example, some languages use delimiters as in the following grammar:

```
<if-stmt> ::= if <expr> then <stmt> fi | if <expr> then <stmt> else <stmt> fi
<stmt> ::= <if-stmt> | S1 | S2
```

Alternatively, we could use more non-terminals to “catch” the dangling else:

```
<matched> ::= if <expr> <matched> else <matched> | S1 | S2
<unmatched> ::= if <expr> <stmt> | if <expr> <matched> else <unmatched>
<stmt> ::= <matched> | <unmatched>
```

Here we distinguish between matched if’s (where there is a matching else or the statement does not involve if at all) and if’s without an else.

It is not always possible to remove an ambiguity by restructuring the grammar. A language for which there is no unambiguous grammar is said to be inherently ambiguous. However, there is no algorithm that can tell if a given context free grammar is ambiguous – this is an undecidable problem.

**Attribute Grammars**

A context free grammar as we have seen is sufficient to describe the pure syntax of a language, but correct programs (i.e., accepted by a compiler or interpreter) have stricter requirements than the pure syntax. For example, in the C language, variables must be declared before they are used, and this can be hard (actually impossible) to express with a context free grammar. This language rule is an example of static semantics. It is a rule we can verify by examining the program, i.e., by a static analysis. Rules like this can be expressed formally with an attribute grammar, where we associate attributes with
symbols (e.g., the type of an identifier grammar symbol), using attribute functions to calculate these values. Attribute values can be synthesized or inherited, indicating whether they are obtained by passing information down the syntax tree or up, or whether they are determined outside of the parse tree as intrinsic attributes.

In practice, formal attribute grammars result in such a large set of rules that they are not often used for real programming languages. Rather, compilers implement less formal attribute grammars for issues such as type checking. So, for example, assignment statements could require that the types of the expressions on both sides be the same, and the type of an expression could be determined by rules about how types combine (e.g., the sum of a double and an int is a double). Applying such rules recursively to the parse tree of expressions, where the leaves are atomic elements with defined types, then allows the type of an expression subtree to be determined. This is the same concept as an attribute grammar, but without the formalism.

Dynamic Semantics

So far, we have only talked about static analysis of a program’s correctness, and this only addresses the correct form of the program. We still have yet to address the issue of describing how the program behaves, something any programmer using the language obviously wants to know. One way is through operational semantics, which essentially means we provide an ideal virtual machine to describe how each construct in the language works. That is, we describe its operation by example. There are other more rigorous approaches such as axiomatic and denotational semantics that have the advantage of being able to prove formally the correctness of a program. As with formal attribute grammars, the specification of these semantics often becomes unwieldy for real programming languages.

Axiomatic semantics uses preconditions and postconditions on every statement to indicate constraints on variables in the program. The desired ending state of the program is the postcondition of the last statement, and from that point we work backwards, finding weakest preconditions that can guarantee the postcondition. Finding these preconditions can proceed from axioms or inference. As you might expect, the notation of axiomatic semantics is predicate calculus.

Denotational semantics formally associates functions with the syntactic objects specified by the grammar. This gives a mapping between the objects of the programming language and mathematical objects, which can be more rigorously manipulated. This formalization of the meaning of a program is similar to the informal handling of the abstract syntax tree one sees in practical compilers.

Top-down and bottom-up Parsing

A grammar is a language generator, but a programming language compiler or interpreter needs to have a parser – a parser recognizes the language. It can be shown that for any CFG, we can create a parser that runs in $O(n^3)$ time (where $n$ is the length of the input program), but this amount of time is much too slow for large programs. Fortunately, there are large classes of grammars for which it is possible to build linear time parsers. The most important of these classes are called LL (left-to-right scanning, left-most derivation) and LR (left-to-right scanning, right-most derivation).
LL parsers are top-down (or predictive) parsers. They build the parse tree from the root down by predicting at each step which production will be used to expand the current node after looking at the next token of input. LR parsers are bottom-up parsers, and build the parse tree from the leaves up, recognizing when a set of leaves can be combined as the children of a single parent. Let’s look at these parsers for a simple example:

```plaintext
<csv> ::= val <csv_tail>
<csv_tail> ::= , val <csv_tail> | ;
```

This grammar describes a list of comma separated values terminated by a semi-colon. Here `val` is a terminal token that could be, say A, B, or C. Let’s examine how the different parsers would build the parse tree for the string: `A,B,C;`

The LL parser starts with the root `<csv>`, predicting it will be replaced by `val <csv_tail>` (the only rule), and looks to the input for a `val` token, which it finds (the A). It then predicts (by looking at the input and seeing a comma) that the `<csv_tail>` will be replaced by `, val <csv_tail>`. It then looks for a `val` token, and finds B. Again, it predicts, by peeking at the comma, that `csv_tail` will be replaced by the same rule again, and proceeds in this manner until all input is exhausted, and the tree is built.

The LR parser gets the first token, and sees that it is a `val` (A), so forms a leaf. The next token is a comma, so is another leaf. It continues in this way until it sees a complete right hand side. This happens when it sees the semi-colon, at which point it can form a `<csv_tail>` node, into which it reduces the last leaf. It continues working back through the leaves in this way, reducing and building the parse tree from the bottom up.

Although this example grammar could be parsed top-down or bottom-up, we can see that in the bottom-up case all the input has to be read before the tree can be constructed. In a very large program, this would require too much memory, so this example grammar is not very conducive to bottom-up parsing. By shifting the focus to the front of the list, we can get around this problem, e.g.,

```plaintext
<csv> ::= <csv_prefix> ;
<csv_prefix> ::= <csv_prefix> , val | val
```

However, this grammar can no longer be parsed top-down since we can’t tell the difference between a `<csv_prefix>` and a `val` when we peek ahead, so we don’t know which rule to use. The rule for `<csv_prefix>` is called left recursive since the symbol of the rule also appears as the left most symbol in the rule itself. This property is exactly what makes the grammar desirable for bottom up parsers since it allows for incremental reduction.

Yacc is an LALR (Look Ahead LR) parser generator. It allows disambiguating rules to resolve ambiguities. Thus, you can specify a BNF like grammar to yacc with ambiguities, but use its rules to specify left or right associativity for particular tokens, thus avoiding the need to express that directly in the grammar. A bottom-up parser works by maintaining a stack of the tokens that have been seen. When these tokens constitute a
right hand side of a rule, it can reduce them to the left hand side of the rule. When the parsing is finished, the stack will have one object on it – the root of the tree.

Although parsers for simple languages can be hand crafted, most implementations will use a parser generator that is table driven. Issues of error recovery in parsers can become complex. However, in general, the area of parsers is well known and techniques and tools exist to implement parsers.