Homework 1

Due: Friday, January 19

Review the course policy on homeworks.
To facilitate the grading, turn in the three problems on separate sheets of paper.

1. The input to the following algorithm is a real number $A$ and a positive integer $b$.

   Power($A, b$)
   \[ C \leftarrow A \]
   \[ d \leftarrow b \]
   \[ P \leftarrow 1 \]
   \[ \textbf{while } d > 0 \]
   \[ \textbf{do } \textbf{if } d \text{ is odd } \textbf{then } P \leftarrow P \times C \]
   \[ C \leftarrow C \times C \]
   \[ d \leftarrow \lfloor d/2 \rfloor \]
   \[ \textbf{return } P. \]

   (a) Show that Power($A, b$) correctly computes $A^b$.
   (b) In dealing with real numbers, it is reasonable to assume that a real number takes constant space to represent, and multiplication of two real numbers takes constant time. Assume that $A$ is a real number and that $b$ is an $n$-digit integer. Compare the running time of Power with the “standard” method of computing powers, namely, successively computing, $A^2 = A \times A$, $A^3 = A^2 \times A$, $A^4 = A^3 \times A$, etc.
   (c) Do the same comparison under the assumption that $A$ and $b$ are both $n$-digit integers and multiplication of an $r$-digit integer and an $s$-digit integer costs $O(rs)$.

2. “Hidden line” problems of the following sort arise in graphics applications.
   Given the locations of buildings in a city, devise an algorithm that identifies just the “skyline” of the city.

   The input could be a sequence of triples, where $(l, h, r)$ indicates a building of height $h$ extending from coordinate $l$ to coordinate $r$. The representation of the output is left to you to suggest.

   Target timing: $O(n \log n)$, where $n$ is the number of buildings.

3. Problem 10.1-1